



**ADVANCED GCE**  
**MATHEMATICS (MEI)**  
 Mechanics 4

**4764**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

- Scientific or graphical calculator

**Tuesday 15 June 2010**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (24 marks)

- 1 At time  $t$  a rocket has mass  $m$  and is moving vertically upwards with velocity  $v$ . The propulsion system ejects matter at a constant speed  $u$  relative to the rocket. The only additional force acting on the rocket is its weight.

(i) Derive the differential equation  $m \frac{dv}{dt} + u \frac{dm}{dt} = -mg$ . [4]

The rocket has initial mass  $m_0$  of which 75% is fuel. It is launched from rest. Matter is ejected at a constant mass rate  $k$ .

(ii) Assuming that the acceleration due to gravity is constant, find the speed of the rocket at the instant when all the fuel is burnt. [8]

- 2 A particle of mass  $m$  kg moves horizontally in a straight line with speed  $v$  m s<sup>-1</sup> at time  $t$  s. The total resistance force on the particle is of magnitude  $mkv^{\frac{3}{2}}$  N where  $k$  is a positive constant. There are no other horizontal forces present. Initially  $v = 25$  and the particle is at a point O.

(i) Show that  $v = 4\left(kt + \frac{2}{5}\right)^{-2}$ . [7]

(ii) Find the displacement from O of the particle at time  $t$ . [3]

(iii) Describe the motion of the particle as  $t$  increases. [2]

## Section B (48 marks)

- 3 A uniform rod AB of mass  $m$  and length  $4a$  is hinged at a fixed point C, where  $AC = a$ , and can rotate freely in a vertical plane. A light elastic string of natural length  $2a$  and modulus  $\lambda$  is attached at one end to B and at the other end to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance  $2a$  above C. The string is always vertical. This system is shown in Fig. 3 with the rod inclined at  $\theta$  to the horizontal.

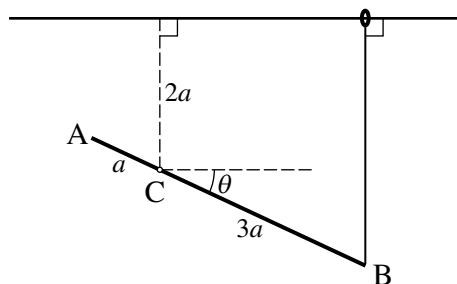


Fig. 3

(i) Find an expression for  $V$ , the potential energy of the system relative to C, and show that  $\frac{dV}{d\theta} = a \cos \theta \left( \frac{9}{2} \lambda \sin \theta - mg \right)$ . [6]

- (ii) Determine the positions of equilibrium and the nature of their stability in the cases

(A)  $\lambda > \frac{2}{9}mg$ , [10]

(B)  $\lambda < \frac{2}{9}mg$ , [4]

(C)  $\lambda = \frac{2}{9}mg$ . [4]

- 4 Fig. 4.1 shows a uniform cone of mass  $M$ , base radius  $a$  and height  $2a$ .

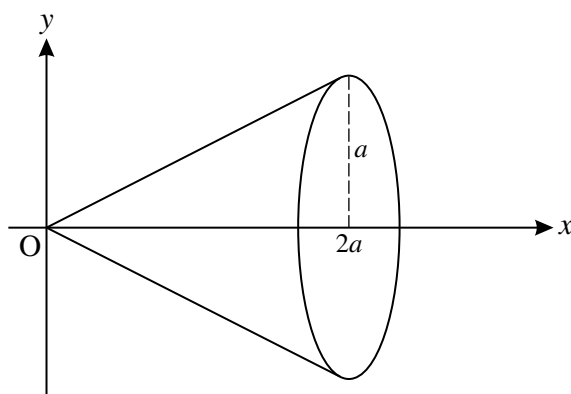


Fig. 4.1

- (i) Show, by integration, that the moment of inertia of the cone about its axis of symmetry is  $\frac{3}{10}Ma^2$ .  
 [You may assume the standard formula for the moment of inertia of a uniform circular disc about its axis of symmetry and the formula  $V = \frac{1}{3}\pi r^2 h$  for the volume of a cone.] [8]

A frustum is made by taking a uniform cone of base radius 0.1 m and height 0.2 m and removing a cone of height 0.1 m and base radius 0.05 m as shown in Fig. 4.2. The mass of the frustum is 2.8 kg.

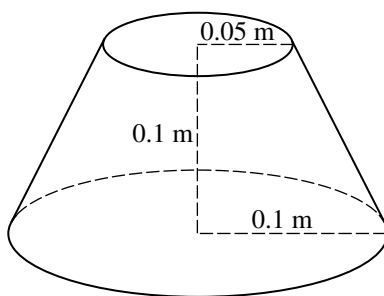


Fig. 4.2

The frustum can rotate freely about its axis of symmetry which is fixed and vertical.

- (ii) Show that the moment of inertia of the frustum about its axis of symmetry is  $0.0093 \text{ kg m}^2$ . [4]

The frustum is accelerated from rest for  $t$  seconds by a couple of magnitude  $0.05 \text{ N m}$  about its axis of symmetry, until it is rotating at  $10 \text{ rad s}^{-1}$ .

- (iii) Calculate  $t$ . [4]

- (iv) Find the position of G, the centre of mass of the frustum. [3]

The frustum (rotating at  $10 \text{ rad s}^{-1}$ ) then receives an impulse tangential to the circumference of the larger circular face. This reduces its angular speed to  $5 \text{ rad s}^{-1}$ .

- (v) To reduce its angular speed further, a parallel impulse of the same magnitude is now applied tangentially in the horizontal plane through G at the curved surface of the frustum. Calculate the resulting angular speed. [5]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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