ADVANCED GCE

MATHEMATICS (MEI) 4764

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

1 At time t a rocket has mass m and is moving vertically upwards with velocity v. The propulsion system ejects matter at a constant speed u relative to the rocket. The only additional force acting on the rocket is its weight.

(i) Derive the differential equation $m\frac{dv}{dt} + u\frac{dm}{dt} = -mg$. **[4]**

The rocket has initial mass m_0 of which 75% is fuel. It is launched from rest. Matter is ejected at a constant mass rate k.

(ii) Assuming that the acceleration due to gravity is constant, find the speed of the rocket at the instant when all the fuel is burnt. **[8]**

2 A particle of mass m kg moves horizontally in a straight line with speed $v\,\mathrm{m\,s^{-1}}$ at time t s. The total resistance force on the particle is of magnitude $mkv^{\frac{3}{2}}$ N where k is a positive constant. There are no other horizontal forces present. Initially $v = 25$ and the particle is at a point O.

(i) Show that $v = 4\left(kt + \frac{2}{5}\right)^{-2}$. **[7]**

(ii) Find the displacement from O of the particle at time t. **[3]**

(iii) Describe the motion of the particle as t increases. **[2]**

Section B (48 marks)

3 A uniform rod AB of mass m and length $4a$ is hinged at a fixed point C, where AC = a, and can rotate freely in a vertical plane. A light elastic string of natural length $2a$ and modulus λ is attached at one end to B and at the other end to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance $2a$ above C. The string is always vertical. This system is shown in Fig. 3 with the rod inclined at θ to the horizontal.

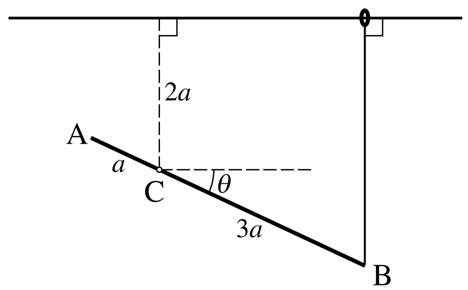

Fig. 3

(i) Find an expression for V, the potential energy of the system relative to C, and show that $\frac{dV}{d\theta} = a\cos\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right)$. **[6]**

(ii) Determine the positions of equilibrium and the nature of their stability in the cases

(A) $\lambda > \frac{2}{9}mg$, **[10]**

(B) $\lambda < \frac{2}{9}mg$, **[4]**

(C) $\lambda = \frac{2}{9}mg$. **[4]**

4 Fig. 4.1 shows a uniform cone of mass M, base radius a and height $2a$.

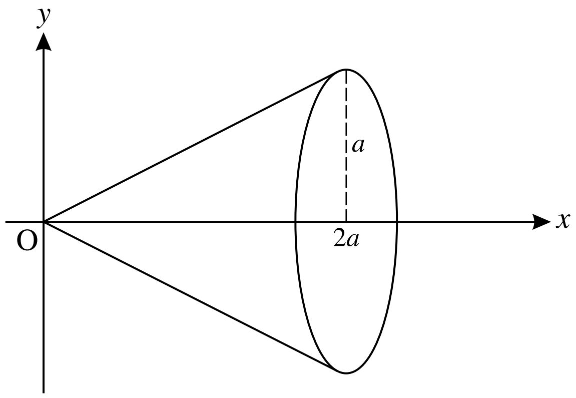

Fig. 4.1

(i) Show, by integration, that the moment of inertia of the cone about its axis of symmetry is $\frac{3}{10}Ma^2$. [You may assume the standard formula for the moment of inertia of a uniform circular disc about its axis of symmetry and the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.] **[8]**

A frustum is made by taking a uniform cone of base radius 0.1 m and height 0.2 m and removing a cone of height 0.1 m and base radius 0.05 m as shown in Fig. 4.2. The mass of the frustum is 2.8 kg.

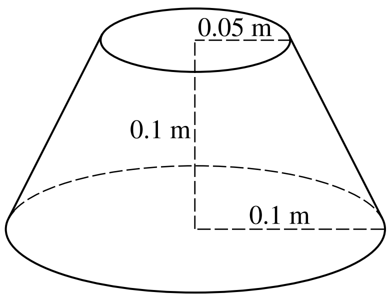

Fig. 4.2

The frustum can rotate freely about its axis of symmetry which is fixed and vertical.

(ii) Show that the moment of inertia of the frustum about its axis of symmetry is $0.0093\,\text{kg}\,\text{m}^2$. **[4]**

The frustum is accelerated from rest for t seconds by a couple of magnitude 0.05 N m about its axis of symmetry, until it is rotating at $10\,\text{rad}\,\text{s}^{-1}$.

(iii) Calculate t. **[4]**

(iv) Find the position of G, the centre of mass of the frustum. **[3]**

The frustum (rotating at $10\,\text{rad}\,\text{s}^{-1}$) then receives an impulse tangential to the circumference of the larger circular face. This reduces its angular speed to $5\,\text{rad}\,\text{s}^{-1}$.

(v) To reduce its angular speed further, a parallel impulse of the same magnitude is now applied tangentially in the horizontal plane through G at the curved surface of the frustum. Calculate the resulting angular speed. **[5]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.